Some calculations regarding the operation of the GPS in flowing space.

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The purpose of this project was to determine whether flowing space (specifically, the resultant anisotropy of light propagation) produces noticeable errors in GPS signal travel times (particularly between signals from satellites at different zenith angles) if the GPS is designed to assume that the speed of light is isotropic and equal to c.

Theory

In Figure 1, we consider a GPS receiver (REC) on the surface of the Earth receiving signals from two GPS satellites: One directly above the receiver (S1) and another on the horizon (S2). The orbital radii of the GPS satellites is $d = 26{,}600\ km$; the radius of the earth is $R = 6371\ km$; and the distance to the satellite on the horizon is $X_d = \sqrt{(d^2 - R^2)}$. The center of the earth is the origin of a Cartesian coordinate system (x,y).

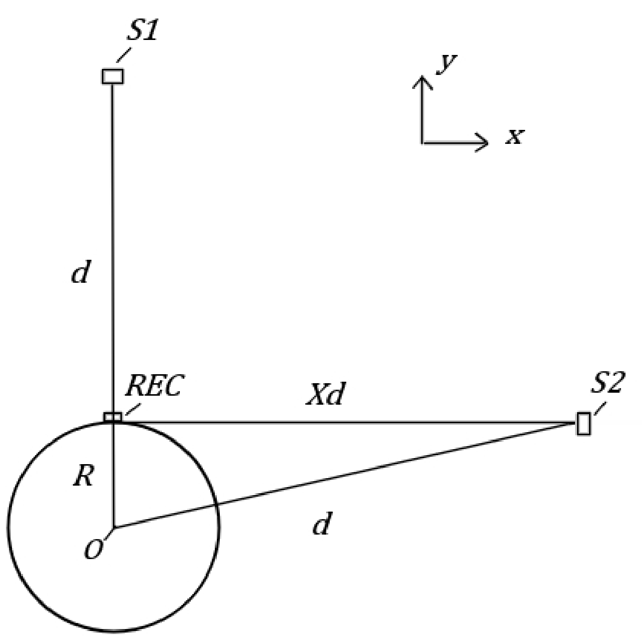

Figure 1 Geometry of the problem. S1 and S2 are GPS satellites communicating with the receiver REC on the Earth.

At the same time, the Earth lies in the center of a radial spatial inflow field given by (1)

$$\boldsymbol{w} = -\sqrt{\frac{2GM}{r}}\,\hat{\boldsymbol{r}}$$

Light moves at c relative to space; that is, if $\boldsymbol{v}$ is the velocity of a light signal relative to Earth, then (2)

$$||\boldsymbol{v} - \boldsymbol{w}|| = c$$

The travel time of the signal from S1 (travelling downwards to Earth) is therefore (3)

$$T1 = \int_R^d \frac{dr}{c + \sqrt{\frac{2GM}{r}}}$$

To find the travel time of the signal from S2, require that $\boldsymbol{v}$ is parallel to the -x-axis at all times; that is,

$$\boldsymbol{v} = -v\,\hat{\boldsymbol{x}}$$

where $v \geq 0$.

From these equations we find

$$v = w_x + \sqrt{c^2 - w_y^2}$$

where

$$w_x = w\cos\theta = w\frac{x}{r} = \sqrt{\frac{2GM}{r}}\frac{x}{\sqrt{x^2+R^2}} = \sqrt{2GM}\,\frac{x}{(x^2+R^2)^{\frac{3}{4}}}$$

$$w_y = w\sin\theta = w\frac{y}{r} = w\frac{R}{\sqrt{x^2+R^2}} = \sqrt{2GM}\frac{R}{(x^2+R^2)^{\frac{3}{4}}}$$

Then (4)

$$T2 = -\int_{X_d}^0 \frac{dx}{v} \quad = \frac{1}{\sqrt{2GM}}\int_0^{X_d} \frac{dx}{\frac{x}{(x^2+R^2)^{\frac{3}{4}}} + \sqrt{\frac{c^2}{2GM} - \frac{R^2}{(x^2+R^2)^{\frac{3}{2}}}}}$$

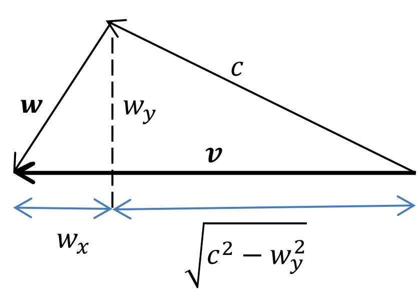

Figure 2 Geometrical origin of the formula for v.

The quantity of interest is the difference between T2 and T1. However, we must first correct T2 for the difference between the geometrical receiver-signal distances for T1 and T2. This difference is

$$\delta L = X_d - (d - R) = \sqrt{d^2 - R^2} - (d - R)$$

The GPS technician is presumably unaware of the Earth's spatial flow field and instead assumes that light travels at c in all directions relative to the Earth. Therefore, he would correct for the difference δL by subtracting $\delta L/c$ from the difference $T2 - T1$ to obtain (5)

$$\delta T = (T2 - \delta L/c) - T1$$

Calculation

To evaluate T1, I integrated equation (3) using Mathematica (it can also be integrated by hand, however). The result was

$$cT1 = 20{,}228.50337\ km$$

For comparison, the travel time $T1_0$ with no flow (that is, if the signal travels at *c*) is

$$cT1_0 = d - r = 20{,}229.0000\ km$$

Calculating T2 is much more complicated. I calculated T2 using two different methods: first, by directly numerically integrating equation (4) in Mathematica. The result was

$$c\delta T = cT2 - \delta L - T1 = 0.0016367\ m = 1.6367\ mm$$

To ensure the precision of this (and all other) calculations, I used the MaxPrecision and MinPrecision settings in Mathematica (with values of 100 and 40, respectively).

For the second method, I calculated the first 3 terms of the Taylor expansion of the integrand $\frac{1}{v}$ for T2 (by hand and by Mathematica to confirm its veracity). These are (6)

$$\frac{1}{v} \approx \frac{1}{c}\left(1 - u\cos\theta + \frac{u^2}{2}(1 + \cos^2\theta) - \frac{u^3}{6}\cos\theta\right)$$

where $u = w/c$ is dimensionless. In Cartesian coordinates, this becomes

$$\frac{1}{v} \approx \frac{1}{c}\left(1 - \frac{\sqrt{2GM}}{c}\frac{x}{(x^2 + R^2)^{\frac{3}{4}}} + \frac{GM}{c^2}\frac{(2x^2 + R^2)}{(x^2 + R^2)^{\frac{3}{2}}} - \frac{\frac{(2GM)^{\frac{3}{2}}}{c^3}x}{(x^2 + R^2)^{5/4}}\right)$$

The terms are the same as in the first equation, in order.

I then integrated the above equation. I was able to integrate each term analytically except for the third one (corr. to u^2). I evaluated each term separately, denoting

$$T2 = T2_0 + T2_1 + T2_2 + T2_3$$

in the same order as the previous equations. I found that $T2_3$ is totally negligible, while the other terms have values

$$T2_0 = \frac{X_d}{c}$$

$$T2_1 = -1.65547 \times 10^{-6} s = -496.641\, m/c$$

$$T2_2 = 4.808 \times 10^{-11} s = 1.442\, cm/c$$

The result is therefore (calculated both by hand and with Mathematica)

$$c\delta T = c(T2 - T1) - \delta L \approx c(T2_1 + T2_2 - T1) - (d - r) = 1.7\, mm$$

Examining the corrections more closely, one notices that by far the greatest part of the correction is from $T2_1$. Moreover, from equation (6) it is clear that $T2_1$is exactly equal to the (approximate) contribution from w_x (that is, the contribution from w along the direction of the signal) alone. Note that

$$\frac{1}{c + w_x} \approx \frac{1}{c}\left(1 - \frac{w_x}{c}\right)$$

The first term here is the 0th (constant) Taylor term (= 1/c) and the second term is the linear Taylor correction(which becomes $T2_1$after integration).

To confirm my suspicion that the contribution from w_y is negligible and the signal travels as if it was only being carried along by w_x, I calculated (rather unnecessarily in view of the above equation)

$$T2' = \int_0^{Xd} \frac{dx}{c + w_x} = -\, 3.8\, mm$$

which, as I suspected, is almost identical to the exact result.

One way of understanding why $w_y$contributes so little (primarily to the very small quadratic Taylor term T22) is to observe that, when one subtracts two quantities a,b geometrically (i.e. as the sides of a right triangle) rather than linearly, if a >> b then the geometric subtraction enormously reduces the effect of subtracting b: $\sqrt{a^2 - b^2} \approx a\left(1 - \frac{b^2}{a^2}\right) = a - b\left(\frac{b}{a}\right)$ vs. $a - b$.

Discussion

The diminutive size of the error induced in the GPS transit times by Flowing Space can be understood as resulting from 5 factors:

1. $w \ll c$. (This is not sufficient, of course, because the difference in T2 between the FS and non-FS cases is 500 meters.)
2. Since $R \sim d/4$, the signal from the horizon satellite S2 propagates mostly *along* (with) the radial flow lines for most of the trip.

3. The vertical ether component subtracts orthogonally/geometrically from v, while the horizontal component adds linearly. Combined with (1), this means that the effect of w_y is reduced by the factor $\frac{w_y}{c} \leq 10^{-4}$ compared to the effect of w_x.
4. The GPS technician corrects for the geometrical signal travel distance as if the signal propagates at c, when in fact it propagates at greater than c for most of the trip. This effectively 'removes' part of the trip where the signal is propagating at approximately or less than c (near the Earth, at r <10,000 km), leaving only the larger portion of the trip where the signal's velocity is more similar to that of the overhead signal.
5. The difference in w between r = d and r = R is only a factor of 2: 5.5 km/s vs. 11.2 km/s.

These same factors will most likely constrain Flowing Space-related errors in VLBI distance measurements to only a few mm as well – again, contrary to what one would expect. And indeed, this webpage www.cpi.com/projects/vlbi.html states that VLBI signal travel distances can be accurate 'down to a few mm'.

Future work

I should repeat the above calculation for GPS satellites at arbitrary angles. To make it quick (and possibly find an analytical expression), I can start by just setting the signal propagation speed equal to $v = c + \boldsymbol{w} \cdot \boldsymbol{v}/||v||$.

Appendix 1: Two-way Earth to Overhead Satellite travel time residual

I also calculated the travel time T_{up} for a signal propagating up against the ether flow, from the receiver to an overhead GPS satellite. I then compared this with $T_{down} = T1$ and $T_0 = (d - R)/c$. I found

$$c(T_{up} - T_0) \approx -c(T_{down} - T_0) \approx 496\, m$$

$$c(|T_{up} - T_0| - |T_{down} - T_0|) \approx 2.4\, mm$$

In "Relativity in the Global Positioning System" (2003) by Neil Ashby, the author states that errors in the two-way (?) travel time from the receiver to the GPS of 2 cm or less "can be neglected for most purposes", and also that the Shapiro delay usually adds up to 2 cm or less (which I confirmed using the formula in the paper with the parameters for an overhead GPS satellite).

Mathematica filename: twowayGPSlinkdelay.nb

Appendix 2: FS-induced errors in the VLBI imaging system

Incidentally, for two VLBI receivers situated 90° apart on the circumference of the Earth, the calculation for the FS-induced difference in signal arrival times is identical to the main calculation of this paper, except with d replaced by the size of the entrainment sphere (i.e. the spatial flow field) of the Earth.

Using the Taylor series expansion program with $d \approx 2.7 \times 10^{10} m$, I found that the difference in signal arrival times due to Flowing Space is

$$c\tau = 2.4\, mm$$

However, the 'exact' calculation produced bizarrely large results – closer to 500 m – which increased by factors of 10 if I increased the distance greatly. This is highly implausible, so I am trying to confirm that the 'exact' program is malfunctioning.

Note that according to the website http://www.cpi.com/projects/vlbi.html, the accuracy of the VLBI system is indeed "a few millimeters", as which is consistent with the flowing space prediction above. Note also that the relative positions of the VLBI receivers in my calculation is probably optimized to produce the largest FS residual.

Mathematica filename: "shapiroGPScalculation(alteredforVLBI).nb"

Appendix 3: The Shapiro Time Delay

To calculate the Shapiro time delay of a signal passing the sun from Earth to Venus and back, one needs only a slightly modified version of the calculation of T2 in the main text. The main difference is the limits of integration: the distance Xd becomes the distance between the Sun and the source/receiver. Also, one needs to do a separate integral for the parts of the trip where the signal is traveling against the flow, because the velocity as a function of distance is different:

$$v_{against\ flow} = -w_x + \sqrt{c^2 - w_y^2}$$

To include the effects of passing through the Earth's and Venus' flow fields, one needs the calculation from Appendix 1 of the time delay for that two-way trip (but with the "GPS satellite radius" extended to the radius of the entrainment sphere). It turns out that those effects are negligible (about 10^{-10} seconds each), as one would expect.

My result:

Reflection from Venus: $\Delta T = 0.00021377$ seconds

Reflection from Mercury: $\Delta T = 0.000201435$

in good agreement with Shapiro's calculation (which was measured to be within $-$ 20% of his calculated $2.0 \times\ 10^{-4}$ s). If I exclude the influence of the Earth and Venus'/Mercury's entrainment fields, I get the same answer.

Update: It looks like Richard Benish has done the same calculation the exact same way as me (but with the reflector being Mars and neglecting the Earth's and Mars' flow fields) in the "Shapiro-Reasenberg

Experiment" section of his article "Light and Clock Behavior in the Space Generation Model of Gravitation", and he got *exactly* the same answer for the time delay as the GR calculation.

NOTE: I also got exactly the same answer if I changed the sign of the Sun's spatial flow velocity – i.e. if I turned the sun into a spatial source rather than a sink.

Mathematica filename: "shapiroGPScalculationModifiedForShapiro.nb"